

Serial-parallel Graphs

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grammar SerialParallel
  nonterminal SP(2), S(0);
  terminal a(2);
  start S;

  S() ::= SP(x,y) [ init ]
  SP(x,y) ::= a(x,y) [ edge ]
           | SP(x,y) SP(x,y) [ parallel ]
           | SP(x,u) SP(u,y) [ serial ]
end

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Note: States $q_0, q_2, q_3, q_4, q_5, q_6$ have conflicts

State $q_0(a, b)$

$S()$	$\rightarrow \cdot SP(a, b)$
$SP(a, b)$	$\rightarrow \cdot SP(a, b) SP(a, b)$
$SP(a, b)$	$\rightarrow \cdot SP(a, n_1) SP(n_1, b)$
$SP(a, b)$	$\rightarrow \cdot a(a, b)$
$SP(a, n_2)$	$\rightarrow \cdot SP(a, n_3) SP(n_3, n_2)$
$SP(a, n_4)$	$\rightarrow \cdot SP(a, n_4) SP(a, n_4)$
$SP(a, n_5)$	$\rightarrow \cdot a(a, n_5)$

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 = b} \rightarrow q_3(n_0, n_1)$$

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_2(n_0, b, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 = b} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_1(n_0, n_1)$$

Conflicts:

- Shift-shift conflict between $a(a, \circ)$ and $a(a, b)$

State $q_1(a, b)$

$SP(a, b)$	$\rightarrow a(a, b) \cdot$ [edge]
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State $q_2(a, b, c)$

$SP(a, b)$	$\rightarrow SP(a, c) \cdot SP(c, b)$
$SP(a, n_1)$	$\rightarrow SP(a, c) \cdot SP(c, n_1)$
$SP(a, c)$	$\rightarrow SP(a, c) \cdot SP(a, c)$
$SP(a, c)$	$\rightarrow \cdot SP(a, c) SP(a, c)$
$SP(a, c)$	$\rightarrow \cdot SP(a, n_2) SP(n_2, c)$
$SP(a, c)$	$\rightarrow \cdot a(a, c)$
$SP(a, n_3)$	$\rightarrow \cdot SP(a, n_4) SP(n_4, n_3)$
$SP(a, n_5)$	$\rightarrow \cdot SP(a, n_5) SP(a, n_5)$
$SP(a, n_6)$	$\rightarrow \cdot a(a, n_6)$
$SP(c, n_7)$	$\rightarrow \cdot SP(c, n_8) SP(n_8, n_7)$
$SP(c, n_9)$	$\rightarrow \cdot SP(c, n_9) SP(c, n_9)$
$SP(c, n_{10})$	$\rightarrow \cdot a(c, n_{10})$
$SP(c, b)$	$\rightarrow \cdot SP(c, n_{11}) SP(n_{11}, b)$
$SP(c, b)$	$\rightarrow \cdot SP(c, b) SP(c, b)$
$SP(c, b)$	$\rightarrow \cdot a(c, b)$

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 = c} \rightarrow q_5(n_0, n_1)$$

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_2(n_0, c, n_1)$$

$$\frac{SP(n_0, n_1)}{n_0 = c, n_1 = b} \rightarrow q_6(n_0, n_1, a)$$

$$\frac{SP(n_0, n_1)}{n_0 = c, n_1 \uparrow} \rightarrow q_4(n_0, b, n_1, a)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 = c} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = c, n_1 = b} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = c, n_1 \uparrow} \rightarrow q_1(n_0, n_1)$$

Conflicts:

- Shift-shift conflict between $a(a, c)$ and $a(a, \circ)$
- Shift-shift conflict between $a(c, b)$ and $a(a, \circ)$
- Shift-shift conflict between $a(c, b)$ and $a(c, \circ)$
- Shift-shift conflict between $a(c, \circ)$ and $a(a, \circ)$

State $q_3(a, b)$

$SP(a, b)$	$\rightarrow SP(a, b) \cdot SP(a, b)$	
$S()$	$\rightarrow SP(a, b) \cdot$	[init]
$SP(a, b)$	$\rightarrow \cdot SP(a, b) SP(a, b)$	
$SP(a, b)$	$\rightarrow \cdot SP(a, n_1) SP(n_1, b)$	
$SP(a, b)$	$\rightarrow \cdot a(a, b)$	
$SP(a, n_2)$	$\rightarrow \cdot SP(a, n_3) SP(n_3, n_2)$	
$SP(a, n_4)$	$\rightarrow \cdot SP(a, n_4) SP(a, n_4)$	
$SP(a, n_5)$	$\rightarrow \cdot a(a, n_5)$	

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 = b} \rightarrow q_5(n_0, n_1)$$

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_2(n_0, b, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 = b} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_1(n_0, n_1)$$

Conflicts:

- Shift-shift conflict between $a(a, \circ)$ and $a(a, b)$

State $q_4(a, b, c, d)$

$SP(a, b) \rightarrow SP(a, c) \cdot SP(c, b)$	
$SP(a, n_1) \rightarrow SP(a, c) \cdot SP(c, n_1)$	
$SP(a, c) \rightarrow SP(a, c) \cdot SP(a, c)$	
$SP(d, c) \rightarrow SP(d, a) SP(a, c) \cdot$	[serial]
$SP(a, c) \rightarrow \cdot SP(a, c) SP(a, c)$	
$SP(a, c) \rightarrow \cdot SP(a, n_2) SP(n_2, c)$	
$SP(a, c) \rightarrow \cdot a(a, c)$	
$SP(a, n_3) \rightarrow \cdot SP(a, n_4) SP(n_4, n_3)$	
$SP(a, n_5) \rightarrow \cdot SP(a, n_5) SP(a, n_5)$	
$SP(a, n_6) \rightarrow \cdot a(a, n_6)$	
$SP(c, n_7) \rightarrow \cdot SP(c, n_8) SP(n_8, n_7)$	
$SP(c, n_9) \rightarrow \cdot SP(c, n_9) SP(c, n_9)$	
$SP(c, n_{10}) \rightarrow \cdot a(c, n_{10})$	
$SP(c, b) \rightarrow \cdot SP(c, n_{11}) SP(n_{11}, b)$	
$SP(c, b) \rightarrow \cdot SP(c, b) SP(c, b)$	
$SP(c, b) \rightarrow \cdot a(c, b)$	

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 = c} \rightarrow q_5(n_0, n_1)$$

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_2(n_0, c, n_1)$$

$$\frac{SP(n_0, n_1)}{n_0 = c, n_1 = b} \rightarrow q_6(n_0, n_1, a)$$

$$\frac{SP(n_0, n_1)}{n_0 = c, n_1 \uparrow} \rightarrow q_4(n_0, b, n_1, a)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 = c} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = c, n_1 = b} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = c, n_1 \uparrow} \rightarrow q_1(n_0, n_1)$$

Conflicts:

- Shift-reduce conflict between $a(c, b)$ and reduce *serial*
- Shift-reduce conflict between $a(c, \circ)$ and reduce *serial*
- Shift-shift conflict between $a(a, \circ)$ and $a(a, c)$
- Shift-shift conflict between $a(a, \circ)$ and $a(c, b)$
- Shift-shift conflict between $a(a, \circ)$ and $a(c, \circ)$
- Shift-shift conflict between $a(c, b)$ and $a(c, \circ)$

State $q_5(a, b)$

$SP(a, b) \rightarrow SP(a, b) \cdot SP(a, b)$	
$SP(a, b) \rightarrow SP(a, b) SP(a, b) \cdot$	[parallel]
$SP(a, b) \rightarrow \cdot SP(a, b) SP(a, b)$	
$SP(a, b) \rightarrow \cdot SP(a, n_1) SP(n_1, b)$	
$SP(a, b) \rightarrow \cdot a(a, b)$	
$SP(a, n_2) \rightarrow \cdot SP(a, n_3) SP(n_3, n_2)$	
$SP(a, n_4) \rightarrow \cdot SP(a, n_4) SP(a, n_4)$	
$SP(a, n_5) \rightarrow \cdot a(a, n_5)$	

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 = b} \rightarrow q_5(n_0, n_1)$$

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_2(n_0, b, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 = b} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_1(n_0, n_1)$$

Conflicts:

- Shift-reduce conflict between $a(a, b)$ and reduce *parallel*
- Shift-reduce conflict between $a(a, \circ)$ and reduce *parallel*
- Shift-shift conflict between $a(a, b)$ and $a(a, \circ)$

State $q_6(a, b, c)$

$SP(a, b) \rightarrow SP(a, b) \cdot SP(a, b)$	
$SP(c, b) \rightarrow SP(c, a) SP(a, b) \cdot$	[serial]
$SP(a, b) \rightarrow \cdot SP(a, b) SP(a, b)$	
$SP(a, b) \rightarrow \cdot SP(a, n_1) SP(n_1, b)$	
$SP(a, b) \rightarrow \cdot a(a, b)$	
$SP(a, n_2) \rightarrow \cdot SP(a, n_3) SP(n_3, n_2)$	
$SP(a, n_4) \rightarrow \cdot SP(a, n_4) SP(a, n_4)$	
$SP(a, n_5) \rightarrow \cdot a(a, n_5)$	

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 = b} \rightarrow q_5(n_0, n_1)$$

$$\frac{SP(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_2(n_0, b, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 = b} \rightarrow q_1(n_0, n_1)$$

$$\frac{a(n_0, n_1)}{n_0 = a, n_1 \uparrow} \rightarrow q_1(n_0, n_1)$$

Conflicts:

- Shift-shift conflict between $a(a, b)$ and $a(a, \circ)$